Abstract

We introduce a new stochastic reserving model tailored to incurred claims triangles, particularly suited for lines of business with long settlement delays. In addition, we propose a parametric bootstrap procedure that delivers substantial computational efficiency gains compared to standard approaches.

Interest in uncertainty quantification has grown steadily in recent years, driven both by compliance requirements and by the need for effective risk management. This development has also encouraged a wider adoption of stochastic reserving methods to quantify the uncertainty of future claims in insurance portfolios.

Let $(X_{ij})_{(i,j)\in\{1,\dots,I\}\times\{0,\dots,J\}}$ denote the incremental claim amounts and

$$\mathcal{D}_I = \{X_{ij} : 1 \le i \le I,\ 0 \le j \le J,\ i+j \le I\}$$

denote the observed incremental claim amounts at the end of calendar period I. Note that the X_{ij}'s can be e.g. claim payments or incurred claims.

It is of interest to find a modeling framework for the predictive distribution of the remaining incremental claim amounts X_{ij} after calendar period I:

$$\sum_{i+j>I} X_{ij}.$$

Such a framework is often desired for the standard case where $X_{ij} \ge 0$ for all (i,j). Here, it can be suitable to use the over-dispersed Poisson (ODP) reserving model from England and Verrall (2002) and apply the non-parametric bootstrap approach implemented by e.g. Gesmann *et al.* (2025) in the `ChainLadder::BootChainLadder`-function.

In practice, however, the reserving actuary may often experience $X_{ij} < 0$ for some lines of business. This can be the case for accident insurance when considering the incremental incurred claims triangle even though chain-ladder might provide a completely reasonable actuarial best estimate. This has been our motivation for introducing a new reserve uncertainty modeling framework consistent with the underlying triangle of such a best estimate.

The ODS Reserving Model

Model 1. Given $\theta \in \mathbb{R}^{I+2J+2}_{\ge 0}$ defined by

$$\theta := (\alpha_1, \dots, \alpha_I, \beta_0^+, \dots, \beta_J^+, \beta_0^-, \dots, \beta_J^-)$$

and $\varphi \in (1,\infty)$, assume that $(X_{ij})_{(i,j)\in\{1,\dots,I\}\times\{0,\dots,J\}}$ are mutually independent with

$$\frac{X_{ij}}{\varphi} \sim \text{Skellam}\left(\frac{\alpha_i\beta_j^+}{\varphi}, \frac{\alpha_i\beta_j^-}{\varphi}\right).$$

Remark 1. Defining $\beta_j := \beta_j^+ - \beta_j^-$ for $j = 0,\dots,J$, the ODS model satisfies

$$\begin{aligned}\mathbb{E}[X_{ij} \mid \theta] &= \alpha_i\,\beta_j,\\ \text{Var}(X_{ij} \mid \theta, \varphi) &= \varphi\,\alpha_i\left(\beta_j + 2\beta_j^-\right) \ \ge\ \varphi\,\alpha_i\beta_j\end{aligned}$$

for $(i,j) \in \{1,\dots,I\}\times\{0,\dots,J\}$.

Remark 2. The ODP model appears as a special case of the ODS model obtained by setting $\beta_j^- = 0$ for $j = 0,\dots,J$. Equivalently, an ODS random variable can be constructed as the difference of two independent ODP random variables with the same dispersion parameter.

Remark 3. For simplicity in this report, the dispersion parameter φ is estimated using Pearson residuals applied to the absolute values of both the observed X_{ij} and the corresponding predictions.

Results

We apply the parametric bootstrap implementation of the ODS model to the synthetic triangle in Table 3. The ODS parameters were calibrated to match the chain-ladder estimates for comparability.

For reference, we compare the numerical results to those obtained from the `BootChainLadder` function based on the ODP reserving model, a widely applied method in practice.

Method	Mean	SD	Median	75^{th} Percentile	90^{th} Percentile	95^{th} Percentile
ODS (`C#`)	16 568 425	3 102 255	16 508 792	18 619 292	20 572 708	21 761 882
BootChainLadder (`R`)	16 504 552	3 317 300	16 490 110	18 700 426	20 704 720	21 693 332

Table 1: Summary statistics of simulated reserves

The two approaches yield very similar reserve estimates and percentiles. Importantly, however, is that the ODS model remains consistent with the negative increments X_{ij} for $j \in \{1, 2\}$ observed in Table 3, while the reference ODP reserving model cannot accommodate such values.

Method	Number of simulations	Median Runtime	Memory Allocation
ODS (`C#`)	1 000	155 ms	16 KB
ODS (`C#`)	10 000	205 ms	16 KB
ODS (`C#`)	100 000	321 ms	16 KB
BootChainLadder (`R`)	1 000	3.2 s	0.64 GB
BootChainLadder (`R`)	10 000	42.5 s	6.28 GB
BootChainLadder (`R`)	100 000	13.1 m	62.8 GB

Table 2: Benchmark results

It appears evident that one can improve the computational performance remarkably by applying this new parametric bootstrap method.

i \ j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	8 951 595	5 046 153	3 382 754	3 835 268	4 624 117	5 276 622	5 846 706	6 392 039	6 772 110	6 904 717	7 014 634	7 035 127	7 095 887	7 109 155	7 114 284	7 159 873	7 196 601	7 196 603	7 196 603	7 196 615	7 196 632	7 196 638	7 196 638	7 196 638	7 196 638	7 196 638	7 196 638	7 196 638	7 196 638	7 196 638
2	11 051 131	6 557 297	4 535 848	4 663 999	5 508 528	6 171 540	6 955 042	7 449 102	7 857 621	8 066 113	8 079 153	8 166 280	8 214 387	8 215 755	8 226 544	8 226 544	8 248 645	8 325 023	8 325 023	8 325 023	8 325 023	8 325 023	8 325 023	8 325 054	8 325 054	8 325 054	8 325 054	8 325 054	8 325 054	
3	8 131 447	5 326 485	2 936 379	3 169 631	4 778 378	6 147 337	6 527 736	7 116 285	7 286 537	7 387 158	7 528 281	7 554 990	7 573 144	7 573 867	7 578 681	7 578 681	7 748 189	7 777 367	7 777 370	7 777 384	7 777 384	7 777 384	7 777 384	7 777 384	7 779 043	7 779 043	7 779 043	7 779 043		
4	10 262 059	7 102 694	4 724 915	4 841 291	5 342 704	5 766 168	6 419 880	6 602 231	6 650 384	6 663 293	6 909 006	6 916 800	6 997 458	6 998 317	6 998 361	7 000 266	7 003 155	7 019 721	7 019 721	7 019 748	7 019 748	7 019 748	7 019 748	7 019 748	7 019 748	7 019 748	7 019 748			
5	10 538 789	7 591 774	5 039 782	5 316 149	6 334 739	7 051 788	7 343 528	7 594 467	7 810 396	7 846 648	7 891 207	7 979 094	7 979 497	7 981 791	8 053 465	8 053 533	8 110 259	8 116 912	8 117 006	8 117 006	8 117 008	8 117 008	8 117 376	8 117 376	8 117 376	8 117 376				
6	9 445 379	5 380 383	2 820 761	3 239 384	4 078 633	4 329 433	5 196 118	5 321 477	5 638 303	5 877 346	6 160 987	6 308 748	6 403 473	6 414 778	6 446 326	6 452 853	6 452 853	6 452 853	6 452 853	6 452 853	6 473 341	6 473 341	6 473 828	6 473 828	6 473 828					
7	10 231 340	7 905 170	5 849 515	5 950 169	6 464 562	7 287 993	7 776 993	8 320 426	8 483 720	9 137 825	9 158 165	9 358 382	9 513 529	9 515 932	9 515 944	9 516 548	9 534 788	9 534 788	9 534 789	9 534 789	9 534 804	9 534 804	9 541 808	9 541 808						
8	10 054 156	7 412 225	5 441 919	5 528 623	6 150 597	6 674 498	7 160 105	7 597 976	7 922 729	8 014 076	8 080 683	8 101 638	8 101 665	8 148 125	8 167 900	8 167 903	8 168 184	8 173 279	8 194 750	8 198 065	8 198 065	8 198 065	8 198 065							
9	10 083 836	7 100 584	2 773 437	3 157 869	4 252 370	5 579 573	6 310 944	6 820 300	7 447 970	7 595 168	7 632 819	7 665 237	7 789 494	7 789 506	7 906 090	7 906 375	7 906 375	7 908 530	7 908 530	7 908 535	7 908 608	7 908 608								
9	10 452 507	6 662 352	4 920 695	5 234 901	6 905 260	7 442 374	8 129 971	8 378 650	8 796 485	8 875 578	8 880 601	10 109 086	9 164 924	9 189 591	9 191 593	9 191 850	9 191 850	9 191 928	9 191 928	9 202 428	9 202 428									
11	9 643 833	5 831 717	2 949 786	3 228 792	4 053 898	4 637 755	5 053 268	5 415 717	5 600 892	5 777 706	5 822 275	5 966 045	6 063 864	6 067 119	6 077 880	6 077 880	6 102 345	6 102 351	6 102 351	6 102 351										
12	9 801 326	6 798 285	4 602 901	5 065 046	6 908 109	7 337 725	8 195 468	8 593 298	8 847 721	9 050 069	9 076 027	9 209 927	9 213 217	9 213 223	9 290 449	9 294 870	9 294 870	9 294 870	9 295 557											
13	9 102 293	6 054 974	3 657 698	3 932 428	5 366 592	6 135 938	6 910 782	7 347 205	7 504 525	7 666 527	7 708 575	7 778 699	7 783 118	7 788 002	7 791 888	7 831 506	7 972 262	7 972 262												
14	10 436 802	7 244 662	4 600 151	4 718 759	5 466 844	6 253 152	6 562 116	7 138 782	7 622 062	7 800 825	8 137 385	8 220 837	8 235 188	8 237 108	8 238 047	8 238 713	8 238 713													
15	12 093 876	9 614 626	7 016 128	7 234 831	7 974 871	8 901 774	9 074 726	9 208 839	9 460 336	9 507 553	9 610 719	9 679 981	9 692 852	9 702 277	9 703 682	9 703 682														
16	9 676 641	7 461 761	5 535 711	5 906 418	6 785 688	7 771 332	8 510 709	8 916 409	9 205 741	9 226 167	9 261 543	9 312 317	9 318 948	9 319 143	9 319 143															
17	9 899 082	4 929 471	2 183 571	2 609 419	3 865 564	4 639 632	5 220 476	5 555 842	5 891 915	5 966 643	5 986 429	6 072 795	6 075 814	6 095 585																
18	9 628 067	6 581 142	4 019 544	4 299 758	6 108 151	7 045 541	7 499 017	8 223 831	8 650 831	8 982 650	9 059 501	9 070 280	9 213 123																	
19	9 047 334	6 186 435	3 361 217	4 024 488	5 472 144	6 002 957	6 427 372	6 609 792	6 664 129	6 757 370	6 953 849	7 227 718																		
20	11 458 177	7 484 858	5 071 566	5 542 022	6 307 883	7 129 400	7 786 078	8 174 210	8 366 392	8 606 511	8 694 726																			
21	10 707 397	6 131 950	3 536 434	3 900 676	4 587 413	5 992 276	7 070 149	7 502 303	7 900 511	8 011 567																				
22	10 369 476	6 674 262	4 623 655	4 925 803	6 512 534	7 860 643	8 712 454	9 262 707	9 489 201																					
23	11 354 803	8 019 112	6 100 185	6 726 389	7 879 139	8 802 965	9 261 664	9 744 075																						
24	11 889 879	7 877 538	4 370 958	4 988 026	6 250 857	7 019 025	7 559 530																							
25	11 665 035	8 538 842	5 007 965	5 397 504	6 593 600	7 868 410																								
26	9 937 940	6 129 223	2 548 694	2 750 784	3 616 102																									
27	12 008 085	7 630 815	5 073 961	5 410 128																										
28	11 686 453	8 335 617	5 503 376																											
29	11 850 154	7 252 701																												
30	11 845 473																													

Table 3: Synthetic quarterly cumulative incurred triangle – Accident insurance

References

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